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SAMPLE PAPER- SET-2 (2024-25)

Time : 03 : 00 Hours | CLASS :– XII-MATHS (041) | M.M. : 80

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4marks each) with sub parts

SECTION - A

1. If A is a 3×4 matrix and B is a matrix such that $A'B$ and AB' are both defined, then the order of the matrix B is :

(a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

2. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is :

(a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$ (c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$

3. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{a} \times \vec{b}| = 1$, then $\vec{a}.\vec{b}$ is equal to

(a) -1 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

4. The value of k for which function $f(x) = \begin{cases} x^2, & x \geq 0 \\ kx, & x < 0 \end{cases}$ is differentiable at x = 0 is :

(a) 1 (b) 2 (c) any real number (d) 0

5. If $y = \frac{\cos x - \sin x}{\cos x + \sin x}$, then $\frac{dy}{dx}$ is :

(a) $-\sec^2\left(\frac{\pi}{4} - x\right)$ (b) $\sec^2\left(\frac{\pi}{4} - x\right)$ (c) $\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$ (d) $-\log\left|\sec\left(\frac{\pi}{4} - x\right)\right|$

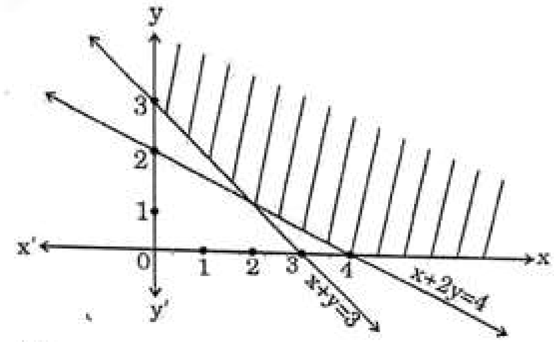
6. What is the product of the order and degree of the differential equation $\frac{d^2y}{dx^2}\sin y + \left(\frac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?

(a) 3 (b) 2 (c) 6 (d) Not defined

7. The value of $\int_0^{\pi/2} \log \tan x \, dx$ is :

(a) $\frac{\pi}{2}$ (b) 0 (c) $-\frac{\pi}{2}$ (d) 1

8. The feasible region of a linear programming problem is shown in the figure below :



Which of the following are the possible constraints ?

(a) $x+2y \geq 4, x+y \leq 3, x \geq 0, y \geq 0$

(b) $x+2y \leq 4, x+y \leq 3, x \geq 0, y \geq 0$

(c) $x+2y \geq 4, x+y \geq 3, x \geq 0, y \geq 0$

(d) $x+2y \geq 4, x+y \geq 3, x \leq 0, y \leq 0$

9. The function $f(x) = |x| - x$ is :

(a) continuous but not differentiable at x = 0.

(b) continuous and differentiable at x = 0.

(c) neither continuous nor differentiable at x = 0.

(d) differentiable but not continuous at x = 0.

10. The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is :

(a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2\log x + \frac{x^3}{3} + C$ (c) $y = \frac{x^2}{2} + C$ (d) $y = 2\log x + \frac{x^2}{2} + C$

11. If f(x) = a (x - cos x) is strictly decreasing in $\mathbb{R}$, then 'a' belongs to

(a) {0} (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$

12. Let A be a 3×3 matrix such that $|adj A| = 64$. Then is equal to :

(a) 8 only (b) -8 only (c) 64 (d) 8 or -8

13. If for two events A and B, $P(A-B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

14. If $\int_0^{2\pi} \cos^2 x\,dx = k\int_0^{\pi/2} \cos^2 x\,dx$, then the value of k is

(a) 4 (b) 2 (c) 1 (d) 0

15. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is

(a) ϕ (b) $\{0\}$ (c) $\{4\}$ (d) $\mathbb{R} - \{4\}$

16. $\int e^{5\log x}\, dx$ is equal to :

(a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$ (c) $5x^4 + C$ (d) $6x^5 + C$

17. The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio :

(a) 1 : 2 internally (b) 2 : 1 internally (c) 3 : 1 internally (d) 3 : 1 externally

18. The events E and F are independent. If P(E) = 0.3 and $P(E \cup F) = 0.5$, then P(E/F)-P(F/E) equals :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{35}$ (d) $\frac{1}{70}$

19. Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$ is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$

Reason (R) : The range of the principal value branceh of $\sin^{-1}(x)$ is

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

20. Assertion (A) : Equation of a line passing through the points (1, 2, 3) and (3, -1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$.

Reason (R) : Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

SECTION - B

21. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

OR

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

22. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$.

23. Find the domain of f(x)= $\sin^{-1}(3x-4x^3)$

24. The total revenue in Rupees received from the sale of x units of a product is given by $R(x)=3x^2+36x+5$. Find the marginal revenue, when x = 5, when by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

25. Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$.

OR

If $y = (\log x)^{\log x}$, x > 1 then find $\frac{dy}{dx}$.

SECTION - C

26. A ladder 5 m long is learning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

27. Solve the differential equation $(\tan^{-1}y-x)dy=(1+y^2)dx$.

28. Consider the following Linear Programming Problem :

Minimise Z = x + 2y

Subject to $2x+y\geq 3, x+2y\geq 6, x, y\geq 0$.

Show graphically that the minimum of Z occurs at more than two points.

29. Find the image of the point (1, 2, 1) with respect to the line $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}$. Also find the equation of the line joining the given point and its image.

30. $\int_0^2 x\sqrt{2-x}\,dx$

OR

Find $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\,dx}{\cos^2 x+4\sin^2 x}$

31. Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male ? Assume that there are equal number of males and females.

OR

Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent survays suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.

A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student in given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1,2,3 \\ 2kx, & \text{for } x = 4,5,6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions:

(i) Express the probability distribution given above in the form of a probability distribution table.

(ii) Find the value of k.

(iii) Find the mean number of hours spent by the student.

SECTION - D

32. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

OR

The area of the region bounded by the line y = mx (m > 0), the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant is $\frac{\pi}{2}$ units. Using integration, find the value of m.

33. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations:

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

OR

If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$ find the value of (a+x)-(b+y).

34. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for, $-1 < x < 1$, prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

OR

Prove that the function f given by $f(x) = |x - 1|, x \in R$ is not differentiable at x = 1.

35. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

SECTION - E

36. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2cm^3/s$. The semi-vertical angle of the conical tank is 45^o.

On the basis of given information, answer the following questions :

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when r ● $2\sqrt{2}$ cm.

(iii) Find the rate at which the wet surface of the coical tank is decreasing at an instant when radiu r ● $2\sqrt{2}$ cm.

OR

(iii) Find the rate of change of height 'h' at an instant when slant height is 4 cm.

37. An organization conducted bike race under two different categories-Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi form two sets B and G with these participants for his college project.

Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions :

(i) How many relations are possible form B to G ?

(ii) Among all the possible relations from B to G, how many functions can be formed from B to G ?

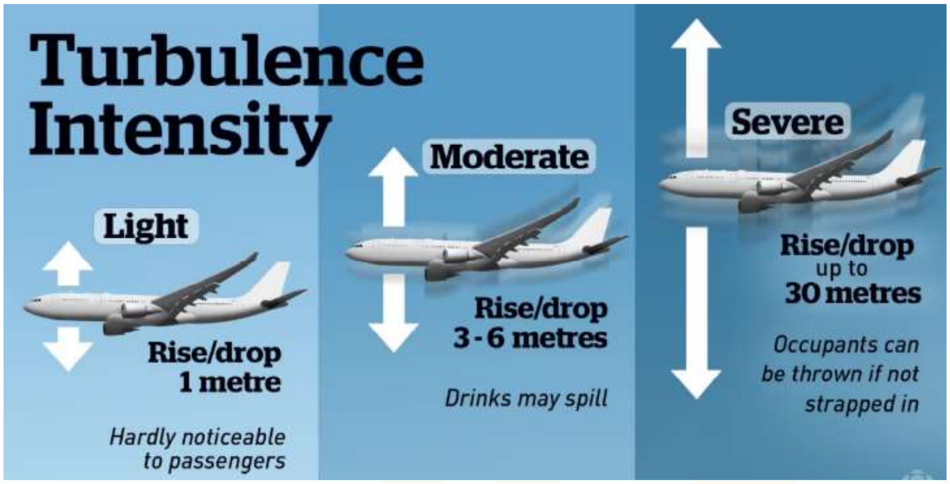
(iii) Let R : B → B be defined by R = {(x, y) : x and y are students of the same sex). Check if R is an equivalence relation.

OR

(iii) A function f : B → G be defined by f = $\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective. Justify your answer.

38. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe tubulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

(i) Find the probability that airplane reached late given that there was a light turbulence in airplane.

(ii) Find the probability that an airplane reached its destination late.

(iii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

OR

(iii) If the airplane reached its destination late, find the probability that it was not due to light turbulence.

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ROUGH WORK