PRINCE ACADEMY
OF HIGHER EDUCATION
[Co-edu. Sr. Sec. School, Affiliated to CBSE, Affiliation No. - 1730387]
Palwas Road, Near Jaipur - Bikaner Bypass Crossing, SIKAR - 332001 (Raj.) INDIA
Mob. : 9610-75-2222, 9610-76-2222
www.princeeduhub.com | E-mail : princeacademy31@gmail.com

SAMPLE PAPER SET - 03 (2024-25)

Time : 03 : 00 Hours **CLASS :– XII-MATHS (041)** **M.M. : 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4marks each) with sub parts

SECTION - A

1. If A = $[a_{ij}]$ is a matrix of order 2×3 such that $a_{ij}=\begin{cases}2 & ,\text{if } i+j \text{ is even}\\ i-2j, & \text{if } i+j \text{ is odd}\end{cases}$ then matrix A is

(a) $\begin{bmatrix}2 & 0 & 2\\ -3 & 2 & 0\end{bmatrix}$ (b) $\begin{bmatrix}2 & 0\\ 2 & -2\\ 2 & 4\end{bmatrix}$ (c) $\begin{bmatrix}2 & -3\\ 0 & 2\\ 2 & -4\end{bmatrix}$ (d) $\begin{bmatrix}2 & -3 & 2\\ 0 & 2 & -4\end{bmatrix}$

2. If A is a matrix of order 3, such that $|\text{adj}A|=5$ which of these could be the value of $|A|$?

(a) 5^2 (b) 5 (c) $\sqrt{5}$ (d) $3\sqrt{5}$

3. Points D(0,2), E(3,1) and F(4,0) are the mid points of sides AB, BC and AC of a triangle ABC, then area of a triangle ABC is

(a) 1 sq unit (b) 2 sq unit (c) 4 sq unit (d) 8 sq unit

4. Which of the following statements is false for the function, $f(x)=[x], x\in R$, where [x] represents greatest integer of $\leq x$?

(a) f(x) is not continuous at x = 3 (b) f(x) is continuous at x = 3
(c) f(x) is not differentiable at x = 3 (d) f(x) is continuous at x = 3.5

5. Let y = $f(x^2)$ and $f'(x)=\sin x$, then value of $\frac{dy}{dx}$ at $x=\sqrt{\frac{\pi}{2}}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\sqrt{2}$ (d) $\sqrt{2\pi}$

6. If $y = \log_2 x^2$, then value of $\frac{dy}{dx}$ at $x = e^{-1}$ is

(a) 2e (b) $2e \log_2 e$ (c) $2e \log^2_e 2$ (d) $\frac{2}{e}$

7. If $y = (\sin x + \cos x)^2$, then $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$ equals

(a) 0 (b) 1 (c) -1 (d) 2

8. In which of these intervals, the function $f(x) = 3x^2 - 4x$, is strictly decreasing ?

(a) $(-\infty,0)$ (b) (0,2) (c) $\left(\frac{2}{3},\infty\right)$ (d) $(-\infty,\infty)$

9. The value of k for which the function, $f(x)=\begin{cases}3x^2, \text{if } x>0\\ kx, \text{if } x<0\end{cases}$ is continuous at x = 0, is

(a) 0 (b) 1 (c) -1 (d) Every real values of k

10. The diameter of a sphere is $\frac{3}{2}(2x+3)$, the rate of change of its surface area with respect to x is

(a) $18\pi(2x+3)$ (b) $\frac{3}{2}$ (c) $9\pi(2x+3)$ (d) $\frac{3}{4}$

11. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 \sin^4 x\,dx$ is

(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi^2}{4}$

12. $\int \sec^2(4-3x)dx$ is equal to

(a) $\cos^2(4-3x)+C$ (b) $-\frac{1}{3}\tan(4-3x)+C$ (c) $-\frac{1}{3}\cos^2(4-3x)+C$ (d) $-\frac{1}{9}\tan(4-3x)+C$

13. A body is generating a sinusoidal wave in a tight horizontal rope by raising and then lowering one end of rope, represented by y = sinx. Then, area of the region bounded by y = sin x between x = 0, $x=\frac{3\pi}{2}$ and x-axis is-

(a) 3 sq units (b) 2 sq units (c) $\frac{3}{2}$sq units (d) 4 sq units

14. In which of the given differential equations is the degree equal to its order ?

(a) $x\frac{dy}{dx}-\frac{d^2y}{dx^2}=0$ (b) $x^2\frac{dy}{dx}+\tan y-\left(\frac{d^2y}{dx^2}\right)^2=0$

(c) $\left(\frac{d^3y}{dx^3}\right)+\sin\left(\frac{dy}{dx}\right)=0$ (d) $x\left(\frac{dy}{dx}\right)^3-\left(\frac{dy}{dx}\right)^2-7=0$

15. For the vector $\left(2\hat{i}-3\hat{j}+4\hat{k}\right)\times\hat{k}$, component vector along x-axis is

(a) $-3\hat{i}$ (b) -3 (c) $2\hat{j}$ (d) $4\hat{k}$

16. Area of a parallelogram whose one side and one diagonal are represented along the vector $3\hat{i}$ and $-5\hat{j}$ is

(a) 4 sq units (b) 8 sq units (c) 15 sq units (d) 7.5 sq units

17. A line makes angles 60°, 60° and 45° with x-axis, y-axis and z-axis respectively, then direction ratios of line are

(a) $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2} \right\rangle$ (b) $\left\langle \sqrt{2}, 1, \sqrt{2} \right\rangle$ (c) $\left\langle 1, 1, \sqrt{2} \right\rangle$ (d) $\left\langle \sqrt{2}, \sqrt{2}, 1 \right\rangle$

18. Line $\vec{r} = 2\hat{j} - 3\hat{k} - \hat{i} + \mu\left(\hat{j} - 3\hat{k} + 5\hat{i}\right)$ is along the direction vector

(a) $2\hat{j} - 3\hat{k} - \hat{i}$ (b) $\hat{j} - 3\hat{k} - \hat{i}$ (c) $5\hat{i} - 3\hat{k} + \hat{j}$ (d) $\hat{i} - 3\hat{k} + 2\hat{j}$

Question numbers 19 and 20 are Assertion-Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the options (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. Assertion (A) : The domain of the function $f(x) = \sin^{-1} 3x$ is $-\frac{1}{3} \le x \le \frac{1}{3}$.

Reason (R) : Principal value of $\cos^{-1} x$ lies in the interval $[0, \pi]$.

20. Assertion (A) : If A and B are two events such that $P(A \cap B) = 0$, then $P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A) \times P(B)}$

Reason (R) : $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

SECTION - B

21. The volume of a cube is increasing at a constant rate. Prove that the rate of increase of its surface area varies inversely as length of the side.

OR

Find the maximum profit that a company can make, if the profit function is given by $P(x) = 105 + 48x - x^2$, where x represents the number of units sold and P represents the profit.

22. Evaluate : $\int x(a - x)^{20} dx$, where a is a constant.

23. Find the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2\tan^{-1}\left(\sqrt{3}\right)$.

OR

If $\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) < \frac{\pi}{3}$, then find the range of values of x.

24. If x = cot t and y = cosec^2 t, then find

(i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$.

25. Find the angle between the lines $\vec{r} = \hat{j} - 3\hat{k} - 3\hat{i} + \mu\left(2\hat{j} - \hat{k} + 4\hat{i}\right)$ and $\frac{x+5}{3} = \frac{2y-2}{1} = \frac{z+1}{-2}$.

SECTION - C

26. If matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1}.

OR

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $A^2 - 4A - 5I = O$

27. If x = a sin t-b cos t and y = a cos t + b sin t, then prove that $\frac{d^2y}{dx^2} = -\left(\frac{x^2+y^2}{y^3}\right)$

28. Evaluate : $\int \frac{1}{1+\tan x} dx$

OR

Evaluate : $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} dx$

29. A random variable X can taken only the values 0, 1, 2, 3. Given that P(X=0) =P(X=1) = p and P(X=2) = P(X=3) such that $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p.

30. Solve the differential equation : $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$.

OR

Solve the differential equation : $\frac{dy}{dx} - 3y\cot x = \sin 2x$.

31. Find the shortest distance between the lines :

$\vec{r} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (\lambda+1)\hat{k}$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

SECTION - D

32. Let T represents the set of triangles in a plane and a relation R in set T is given by $\{(T_1, T_2) \in T \times T : T_1 \text{ is congruent to } T_2\}$. Show that R is an equivalence relation.

OR

Let $N \cup \{0\} \to N \cup \{0\}$ be defined by $f(n) = \begin{cases} n+1, \text{if } n \text{ is even} \\ n-1, \text{if } n \text{ is odd} \end{cases}$

Show that f is a bijective.

33. Find the area of the region bounded by the curve $y^2 = x$ and the lines $y = 2$, $y = 4$ and the y-axis.

OR

Using method of integration find the area of the triangle ABC, coordinates of whose vertices are A(1, -2), B(3, 5) and C(5, 2).

34. Solve the following LPP graphically :

Minimize $Z = 3x + 9y$

subject to constraints

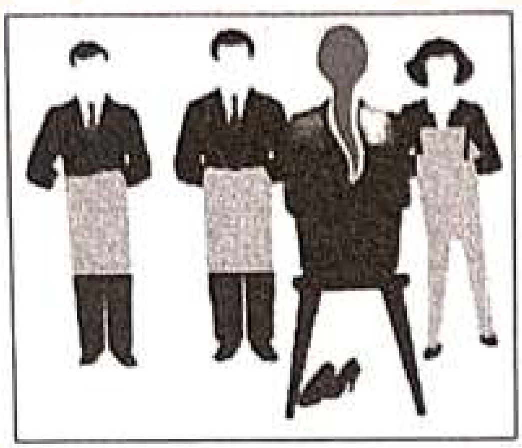
$x + 3y \le 60$, $x + y \ge 10$, $x \le y, x \ge 0, y \ge 0$.

35. Find the vector and cartesian equations of the line pasing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

SECTION - E

In this section, there are 3 case studybased question of 4 marks each.

36. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other.



Based on the above information, answer the following questions :

(i) What is the probability that at least one of them is selected ?

(ii) Find $(G|\overline{H})$ where G is the event of jaspreet's selection and $\overline{H}$ denotes the event that Rohit is not selected.

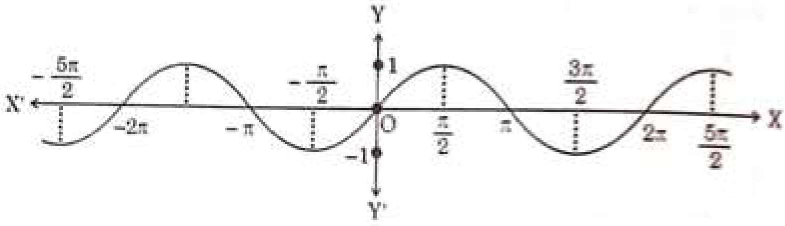
(iii) Find the probability that exactly one of them is selected.

OR

(iii) Find the probability that exactly two of them are selected.

37. If a function $f : X \to Y$ defined as f(x) = y is one-one and onto, then we can define a unique function $g : Y \to X$ such that g(y) = x, where $x \in X$ and y=f(x), $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be deined from set A to [-1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [-1, 1] to A.

(i) If A is the interval other than principal value branch, give an example of one such interval.

(ii) If $\sin^{-1}(x)$ is defined from [-1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$.

(iii) (a) Draw the graph of $\sin^{-1} x$ from [-1, 1] to its principal value branch.

OR

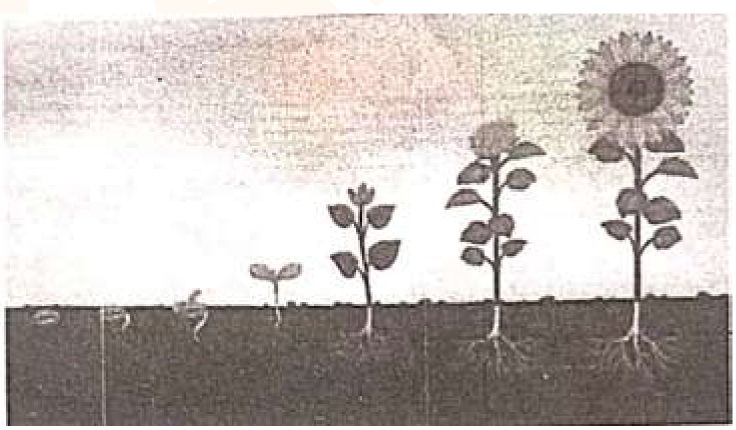
(b) Find the domain and range of $f(x)=2\sin^{-1}(1-x)$.

38. In an agricultural institute, scienctists do experiments with varieties of seeds to grow them in different enviroments to produce healthy plants and get more yield.

A scientist observed that a particular seed grew very fast after germination. He had recorded growth of plant since germination and he said that its growth can be defined by the function.

$$f(x) = \frac{1}{3}x^3 - 4x^2 + 15x + 2, 0 \le x \le 10$$

where x is the number of days the plant is exposed to sunlight.



On the basis of the avove information, answer the following questions :

(i) What are the critical points of the function f(x) ?

(ii) Using second derivative test, find the minimum value of the function.

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ROUGH WORK